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Inverse problem of coupled heat and moisture transport for prediction of moisture distributions in an annular cylinder

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Abstract

This study presents a means of solving the inverse boundary value problem of coupled heat and moisture transport in an annular cylinder. While knowing the temperature history at any point of the body, the boundary time-varying moisture flux can be computed and subsequently the moisture distribution in the body can be determined as well. The surface moisture flux is then determined to minimize the sum of squares of the deviation between the calculated and measured temperatures in the body. The accuracy of the inverse analysis is examined by using simulated exact and inexact measurements obtained on the surface and in an interior location of the cylinder. Numerical results demonstrate that excellent estimations on the moisture distributions can be obtained for all the test cases considered here. The proposed method is highly promising for designing a moisture sensor for some solids.

Nomenclature

C dimensionless concentration of moisture
D equivalent diffusion coefficient of moisture
J functional
L equivalent diffusion coefficient of temperature
p direction of descent
q moisture flux
r dimensionless radius
r_1 dimensionless inner radius of cylinder ($r_1 = 1.0$)
r_2 dimensionless interface radius of two-layer cylinder ($r_2 = 1.6$)
r_3 dimensionless outer radius of cylinder ($r_3 = 2.0$)
t dimensionless time
T dimensionless temperature.

Greek symbols
β step size
Δ small variation quality
ε very small value
λ coupling coefficient due to heat conduction
v coupling coefficient due to moisture migration
σ standard deviation
ω random variable.

Subscripts
f final state
i $i = 1, 2$, denote inner and outer layer separately
0 reference state.

Superscripts
K iterative number
— dimensional quantities.

1. Introduction

To control precisely and reliably a process of moisture diffusion in many technologies such as some materials during storage and in use, and its drying process, it is necessary to know the moisture distribution in the materials. However, only a limited portion of the literature is concerned with the field of moisture distribution. Because experimental determination of the surface and interior moisture distribution is rather difficult, Trofimov

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et al. [1] utilized the measured weight and inverse method to obtain the surface moisture of a body for a problem of moisture diffusion. Physically, there exists a coupling effect between temperature and moisture transport in most materials. The coupling effect is significant for some porous and composite materials. In this work, we investigate the problem of coupled heat and moisture transport in an infinitely annular cylinder where the outer boundary time-varying moisture flux is estimated by the time history of measured temperature at an interior point or on the surface.

In this paper, an analytical method developed in our previous work is applied to the direct problem [2, 3]. A measured location can be chosen on the surface or in the interior of the cylinder. When a thermocouple is located in the cylinder's interior, to obtain an analytical solution for the direction problem, we regard the problem as a two-layer cylinder problem; the two layer cylinders have identical materials properties and its interface coincide with the point of insertion of the thermocouple. The moisture and temperature distributions can be determined using the analytical method. However, when the sensor is located on the surface of the cylinders, then the above two-layer problem becomes a one-layer problem; the moisture and temperature distribution can be obtained as well. In addition, the conjugate gradient method [4, 5] is employed to estimate the surface moisture flux, thereby allowing for the moisture distributions to be predicted for various times.

2. Direct problem

Consider an infinitely long annular cylinder subjected to unknown moisture flux $q(\bar{t})$ on the outer boundary surface, as shown in Fig. 1. The cylinder has inner and outer radii $\bar{r}_1$ and $\bar{r}_3$, respectively. The governing equation an be expressed as follows [2]:

$$L\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}\right)=\frac{\partial}{\partial \bar{t}}(\bar{T}-v\bar{C}) \tag{1a}$$

$$D\left(\frac{\partial^2 \bar{C}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}}\right)=\frac{\partial}{\partial \bar{t}}(\bar{C}-\lambda\bar{T}) \tag{1b}$$

where λ and v are the coupling coefficients; L and D are the equivalent diffusion coefficients of temperature and moisture, respectively; and $\bar{T}$ and $\bar{C}$ are temperature and moisture concentration, respectively.

The associated hygrothermal boundary and initial conditions may be the following:

$$\bar{T}(\bar{r}_1,\bar{t})=\bar{T}_0, \quad \bar{t}>0 \tag{2a}$$

$$\bar{C}(\bar{r}_1,\bar{t})=\bar{C}_0, \quad \bar{t}>0 \tag{2b}$$

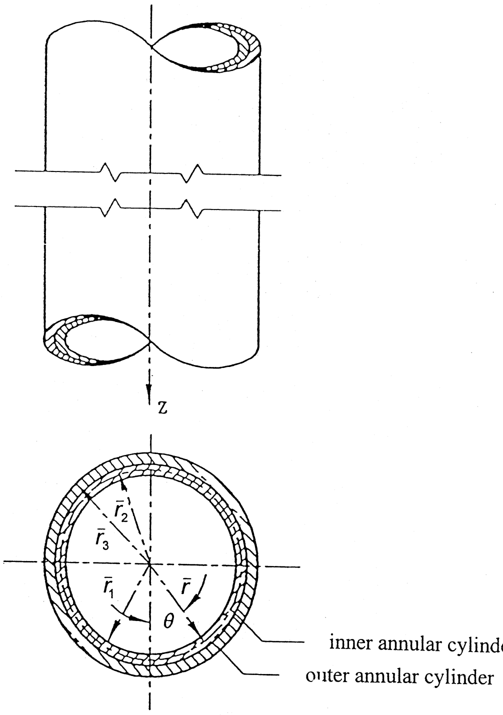

Fig. 1. Infinitely long annular cylinder with a thermocouple located at $\bar{r}=\bar{r}_2$ or $\bar{r}=\bar{r}_3$.

$$\frac{\partial \bar{T}(\bar{r}_3,\bar{t})}{\partial \bar{r}}=0, \quad \bar{t}>0 \tag{2c}$$

$$D\frac{\partial \bar{C}(\bar{r}_3,\bar{t})}{\partial \bar{r}}=\bar{q}(\bar{t}), \quad \bar{t}>0 \tag{2d}$$

$$\bar{T}(\bar{r},0)=\bar{T}_0 \quad \bar{r}_1<\bar{r}<\bar{r}_3 \tag{3a}$$

$$\bar{C}(\bar{r},0)=\bar{C}_0 \quad \bar{r}_1<r<\bar{r}_3. \tag{3b}$$

In order to reduce the number of parameters that have to be specified for the numerical work, dimensionless variables and parameters are defined as

$$r=\frac{\bar{r}}{\bar{r}_1} \quad r_1=\frac{\bar{r}_1}{\bar{r}_1} \quad r_2=\frac{\bar{r}_2}{\bar{r}_1} \quad r_3=\frac{\bar{r}_3}{\bar{r}_1} \quad t=\frac{D\bar{t}}{\bar{r}_1^2}$$

$$q=\frac{\bar{r}_1\bar{q}}{v(\bar{C}_{\rm f}-\bar{C}_0)D} \quad T=\frac{\bar{T}-\bar{T}_0}{v(\bar{C}_{\rm f}-\bar{C}_0)} \quad C=\frac{\bar{C}-\bar{C}_0}{\bar{C}_{\rm f}-\bar{C}_0}$$

where $\bar{T}_0$ and $\bar{C}_0$ are the reference temperature and moisture concentration, respectively, $\bar{C}_{\rm f}$ is the final equilibrium moisture concentration.

When a thermocouple is placed in the cylinder's interior, to employ the decoupled technique [2] and to obtain the analytical solution, we consider the cylinder as a two-layer annular cylinder, which have identical material properties; in addition the interface of two layers in perfect moisture and heat contact coincide with the point of insertion of the thermocouple (i.e. at $r=r_2$). However, when the sensor is located on the cylinder's surface, then the above two-layer problem becomes a one-layer problem. The two cases can be solved by the same scheme. Therefore the following descriptions are for the two-layer problem only.

We then rewrite Eqs. (1)–(3) as

$$\frac{L}{D}\left(\frac{\partial^2 T_i}{\partial r^2}+\frac{1}{r}\frac{\partial T_i}{\partial r}\right)=\frac{\partial}{\partial t}(T_i-C_i)\quad i=1,2 \tag{4a}$$

$$\frac{\partial^2 C_i}{\partial r^2}+\frac{1}{r}\frac{\partial C_i}{\partial r}=\frac{\partial}{\partial t}(C_i-v\lambda T_i)\quad i=1,2 \tag{4b}$$

$$T_1(r_1,t)=0 \tag{5a}$$

$$C_1(r_1,t)=0 \tag{5b}$$

$$T_1(r_2,t)=T_2(r_2,t) \tag{5c}$$

$$\frac{\partial T_1(r_2,t)}{\partial r}=\frac{\partial T_2(r_2,t)}{\partial r} \tag{5d}$$

$$\frac{\partial T_2(r_3,t)}{\partial r}=0 \tag{5e}$$

$$\frac{\partial C_2(r_3,t)}{\partial r}=q(t) \tag{5f}$$

$$T_i(r,0)=0\quad i=1,2 \tag{6a}$$

$$C_i(r,0)=0\quad i=1,2. \tag{6b}$$

An analytical solution of temperature and moisture concentration for the above problem has been developed in our previous work (refer to the Appendix).

3. Sensitivity problem

It is assume that when $q(t)$ undergoes a variation $\Delta q(t)$, the temperature $T(r,t)$ and the moisture concentration $C(r,t)$ change by an amount $\Delta T(r,t)$ and $\Delta C(r,t)$, respectively. We replace q by $q+\Delta q$, T by $T+\Delta T$, and C by $C+\Delta C$ in the direct problem and subtract from it the original problem Eqs. (4)–(6). Finally, the following sensitivity problem is obtained.

$$\frac{L}{D}\left(\frac{\partial^2 \Delta T_i}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta T_i}{\partial r}\right)=\frac{\partial}{\partial t}(\Delta T_i-\Delta C_i)\quad i=1,2 \tag{7a}$$

$$\frac{\partial^2 \Delta C_i}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta C_i}{\partial r}=\frac{\partial}{\partial t}(\Delta C_i-v\lambda \Delta T_i)\quad i=1,2 \tag{7b}$$

$$\Delta T_1(r_1,t)=0 \tag{8a}$$

$$\Delta C_1(r_1,t)=0 \tag{8b}$$

$$\Delta T_1(r_2,t)=\Delta T_2(r_2,t) \tag{8c}$$

$$\frac{\partial \Delta T_1(r_2,t)}{\partial r}=\frac{\partial \Delta T_2(r_2,t)}{\partial r} \tag{8d}$$

$$\frac{\partial \Delta T_2(r_3,t)}{\partial r}=0 \tag{8e}$$

$$\frac{\partial \Delta C_2(r_3,t)}{\partial r}=\Delta q(t) \tag{8f}$$

$$\Delta T_i(r,0)=0\quad i=1,2 \tag{9a}$$

$$\Delta C_i(r,0)=0\quad i=1,2. \tag{9b}$$

4. Adjoint problem and gradient equation

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[q(t)]=\int_0^{t_f}[T(r_2,t)-Y(r_2,t)]^2\,\mathrm{d}t \tag{10}$$

where T is the estimated temperature at the measured location $r=r_2$ in the cylinder; and Y is the measured temperature at the point.

To derive the adjoint problem for $q(t)$, Eqs. (4a) and (4b) are multiplied by the Lagrange multiplier $\psi(r,t)$ and $\phi(r,t)$, respectively; and the resulting expressions are integrated over the time domain, then the result is added to the right-hand side of Eq. (10) to yield the following form:

$$\begin{aligned}J[q(t)]=&\int_0^{t_f}[T(r_2,t)-Y(r_2,t)]^2\,\mathrm{d}t\\&+\frac{L}{D}\sum_{i=1}^{2}\int_{r_1}^{r_{i+1}}\int_0^{t_f}\psi_i(r,t)\cdot\left[r\left(\frac{\partial^2 T_i}{\partial r^2}+\frac{1}{r}\frac{\partial T_i}{\partial r}\right)\right.\\&\left.-\frac{\partial}{\partial t}(T_i-C_i)\right]\mathrm{d}t\,\mathrm{d}r\\&+\sum_{i=1}^{2}\int_{r_1}^{r_{i+1}}\int_0^{t_f}\phi_i(r,t)\left[r\left(\frac{\partial^2 C_i}{\partial r^2}+\frac{1}{r}\frac{\partial C_i}{\partial r}\right)\right.\\&\left.-\frac{\partial}{\partial t}(C_i-v\lambda T_i)\right]\mathrm{d}t\,\mathrm{d}r.\end{aligned} \tag{11}$$

The variation ΔJ is obtained by perturbing $T_i(r,t)$ by $\Delta T_i(r,t)$ and $C_i(r,t)$ by $\Delta C_i(r,t)$ in Eq. (11) and then by subtracting from the resulting expression the original Eq. (11), we thus find

$$\Delta J = 2\int_0^{t_f}[T(r_2,t) - Y(r_2,t)]$$

$$\times \Delta T(r_2,t)\mathrm{d}t + \frac{L}{D}\sum_{i=1}^{2}\int_{i=1}^{2}\int_0^{t_f}\psi_i(r,t)$$

$$\cdot\left[r\left(\frac{\partial^2 \Delta T_i}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta T_i}{\partial r}\right) - \frac{\partial}{\partial t}(\Delta T_i - \Delta C_i)\right]\mathrm{d}t\,\mathrm{d}r$$

$$+\sum_{i=1}^{2}\int_{r_1}^{r_{i+1}}\int_0^{t_f}\phi_i(r,t)\left[r\left(\frac{\partial^2 \Delta C_i}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta C_i}{\partial r}\right)\right.$$

$$\left.-\frac{\partial}{\partial t}(\Delta C_i - v\lambda\Delta T_i)\right]\mathrm{d}t\,\mathrm{d}r. \tag{12}$$

The double integral terms in this equation are integrated by parts; the boundary and initial conditions of the sensitivity problem given by Eqs. (7)–(9) are utilized and then ΔJ is allowed to zero. The vanishing of the boundary value problem containing ΔT_i and ΔC_i leads to the following adjoint problem.

$$\frac{L}{D}\left(\frac{\partial^2\psi_i}{\partial r^2} + \frac{1}{r}\frac{\partial\psi_i}{\partial r}\right) + \frac{\partial\psi_i}{\partial t} - v\lambda\frac{\partial\phi_i}{\partial t} = 0 \quad i = 1, 2 \tag{13a}$$

$$\frac{\partial^2\phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_i}{\partial r} + \frac{\partial\phi_i}{\partial t} - \frac{\partial\psi}{\partial t} = 0 \quad i = 1, 2 \tag{13b}$$

$$\psi_1(r_1,t) = 0 \tag{14a}$$

$$\phi_1(r_1,t) = 0 \tag{14b}$$

$$\psi_1(r_2,t) = \psi_2(r_2,t) \tag{14c}$$

$$\phi_1(r_2,t) = \phi_2(r_2,t) \tag{14d}$$

$$\frac{\mathrm{d}\psi_1(r_2,t)}{\mathrm{d}r} - \frac{\mathrm{d}\psi_2(r_2,t)}{\mathrm{d}r} = \frac{D}{Lr_2}[T(r_2,t) - Y(r_2,t)] \tag{14e}$$

$$\frac{\mathrm{d}\phi_1(r_2,t)}{\mathrm{d}r} = \frac{\mathrm{d}\phi_2(r_2,t)}{\mathrm{d}r} \tag{14f}$$

$$\frac{\mathrm{d}\psi_2(r_3,t)}{\mathrm{d}r} = 0 \tag{14g}$$

$$\frac{\mathrm{d}\phi_2(r_3,t)}{\mathrm{d}r} = 0 \tag{14h}$$

$$\psi_i(r,t_f) = 0 \quad i = 1, 2 \tag{15a}$$

$$\phi_i(r,t_f) = 0 \quad i = 1, 2. \tag{15b}$$

Finally, the only integral term is left

$$\Delta J = \int_0^{t_f}\phi_2(r_3,t)r_3\Delta q(t)\,\mathrm{d}t. \tag{16}$$

From the definition used in [5], we have

$$\Delta J = \int_0^{t_f} J'(r,t)\Delta q(t)\,\mathrm{d}t \tag{17}$$

where $J'(r,t)$ is the gradient of the functional $J(q)$, a comparison of Eqs. (16) and (17) leads to the following form:

$$J'(t) = \phi_2(r_3,t)r_3. \tag{18}$$

5. Conjugate gradient method for minimization

The unknown function $q(t)$ can be determined by a procedure based on the minimization of the functional $J(q)$ with an iterative scheme.

$$q^{K+1} = q^K - \beta^K p^K, \quad K = 0, 1, 2, \ldots \tag{19}$$

where β^K is the step size in going from iteration K to iteration $K+1$ and p^K is the direction of descent, as

$$p^K = J'^K + \gamma^K p^{K-1}, \quad K = 1, 2, 3, \ldots \tag{20}$$

which γ^K is the conjugate coefficient, and is determined from

$$\gamma^K = \frac{\int_0^{t_f}[J'^K(t)]^2\,\mathrm{d}t}{\int_0^{t_f}[J'^{K-1}(t)]^2\,\mathrm{d}t} \quad \text{with} \quad \gamma^0 = 0$$

$$\text{and} \quad K = 1, 2, 3, \ldots \tag{21}$$

The functional $J\,[q(t)]$ for iteration $K+1$ is obtained by rewriting Eq. (10) as

$$J(q^{K+1}) = \int_0^{t_f}[T(q^K - \beta^K p^K) - Y(r_2,t)]^2\,\mathrm{d}t \tag{22}$$

and by a Taylor series expansion, we obtain

$$J(q^{K+1}) = \int_0^{t_f}[T(q^K) - \beta^K\Delta T(p^K) - Y(r_2,t)]^2\,\mathrm{d}t. \tag{23}$$

The sensitivity function $\Delta T(p^K)$ is taken as the solution of Eqs. (7)–(9) at the measured position $r = r_2$ by letting $\Delta q = p^K$. The search step size β^K can be determined by minimizing the function given by Eq. (23) with respect to β. After rearrangement, the following expression is obtained

$$\beta^K = \frac{\int_0^{t_f} \Delta T(p^K)[T(q^K) - Y]\,dt}{\int_0^{t_f} [\Delta T(p^K)]^2\,dt}. \tag{24}$$

6. Stopping criterion

If the problem contains no measurement errors, the convergence condition for the minimization of the criterion is:

$$J(q^{K+1}) < \varepsilon \tag{25}$$

where ε is related to the accuracy of the direct problem solution.

For measured temperature Y perturbed by an additive random error with a standard deviation σ, the following expression is obtained for ε

$$\varepsilon = \sigma^2 t_f. \tag{26}$$

Then, the stopping criterion is given by Eq. (25) with ε determined from Eq. (26).

7. Results and discussions

In order to demonstrate the accuracy and efficiency of the present method, we consider the exact value of the moisture flux $q*(t)$ as

$$q*(t) = 0.3\sin(2\pi t) + 0.25\sin(4\pi t) + 3t(1.1 - t).$$

The material properties and radius of the cylinder structure are listed as follows [6]:

$v = 0.5\,\text{cm}^3\,\text{K}^{-1}\,\text{g}^{-1}$
$\lambda = 0.5\,\text{g cm}^{-3}\,\text{K}^{-1}$
$L = 7.78 \times 10^{-6}\,\text{cm}^2\,\text{h}^{-1}$
$D = 7.78 \times 10^{-7}\,\text{cm}^2\,\text{h}^{-1}$
$r_1 = 1.0,\ r_2 = 1.6,\ r_3 = 2.0.$

To compare the results for situations involving random measurement errors, the simulated inexact measurement data Y can be expressed as

$$Y = Y_{\text{exact}} + \omega\sigma$$

where Y_{exact} is the temperature of the direct problem with the exact moisture flux of $q(t)$, σ is the standard deviation of the measurement, and ω is a random variable within $-2.576 \sim 2.576$ for a 99% confidence bounds.

The total measurement time is chosen as $t_f = 1.0$ and measurement time step is taken 0.01; a thermocouple can be located either on the outer surface or in interior (i.e. $r = r_2$) of the annular cylinder.

The estimated moisture flux of $q(t)$, obtained at the 30th iteration and initial guesses $q^0 = 0.4$ with measurement errors $\sigma = 0.0$, 0.005, and 0.01 are shown in Figs. 2–4, respectively. For a temperature of unity and 99% confidence, these standard deviations correspond to measurement error of 0.0, 1.35 and 2.6%, respectively. As expected, increases in the measurement errors cause decreases in the accuracy of the inverse solution. Figs. 3 and 4 also reveal that good estimation for the moisture flux by interior measurement, since measuring the temperature at the two-layer interface can reduce the sensitivity of measured errors for satisfying the interfacial conditions at every iteration; while the computational time required is about four times longer than that of the measurement on the boundary. Almost the same results as in Fig. 4 also appear in Fig. 5 except near the final time $t = t_f = 1.0$, which is estimated at the 30th iteration and the initial guesses $q^0 = 10^{-3}$ with $\sigma = 0.01$. This indicate that the effect of an initial guess on estimating moisture flux is small.

The relative error between the exact and estimated value for moisture flux and moisture on the outer surface are listed in Table 1 and such an error is defined as

$$\varepsilon_q = \left\{ \int_0^{t_f} [q^*(t) - q(t)]^2\,dt \Big/ \int_0^{t_f} q^{*2}(t)\,dt \right\}^{1/2}$$

$$\varepsilon_C = \left\{ \int_0^{t_f} [C^*(t) - C(t)]^2\,dt \Big/ \int_0^{t_f} C^{*2}(t)\,dt \right\}^{1/2}$$

where $q^*(t)$ and $C^*(t)$ are the exact moisture flux and the exact moisture concentration on the outer surface, respectively.

The data in Table 1 are obtained at the 30th iteration and the initial guesses $q^0 = 0.4$ with various measurement errors σ. The table lists not only the relative error for the measurement locations in an interior and on the surface, but also illustrates that the accuracy estimated for moisture concentration is better than that for moisture flux under the same measurement error. This implies that the inverse method, which can predict the moisture concentration instead of the moisture flux, may be better, since the moisture prediction is less sensitive to the measurement error. Therefore, the prediction of moisture distribution for various times are in excellent agreement with the exact result as shown in Figs. 6 and 7, which obtained with measurement error $\sigma = 0.01$ at the 30th iteration and the initial guesses $q^0 = 0.4$ and 10^{-3}, respectively. The two figures also show that excellent estimated results can be obtained even for the problem even involving the measurement error and giving different initial guess.

8. Conclusion

This study presents a method to solve the inverse boundary value problem of coupled heat and moisture

Table 1
Comparison of accuracy of the results by interior and surface measurement at initial guesses $q_0 = 0.4$ and 30th iteration with various measured errors σ

Measured location	$\sigma = 0.0$		$\sigma = 0.005$		$\sigma = 0.01$	
	ε_q	ε_C	ε_q	ε_C	ε_q	ε_C
Interior measurement ($r = 1.6$)	0.00826	0.00094	0.01155	0.00123	0.01825	0.00218
Surface measurement ($r = 2.0$)	0.00266	0.00020	0.02390	0.00211	0.03937	0.00412

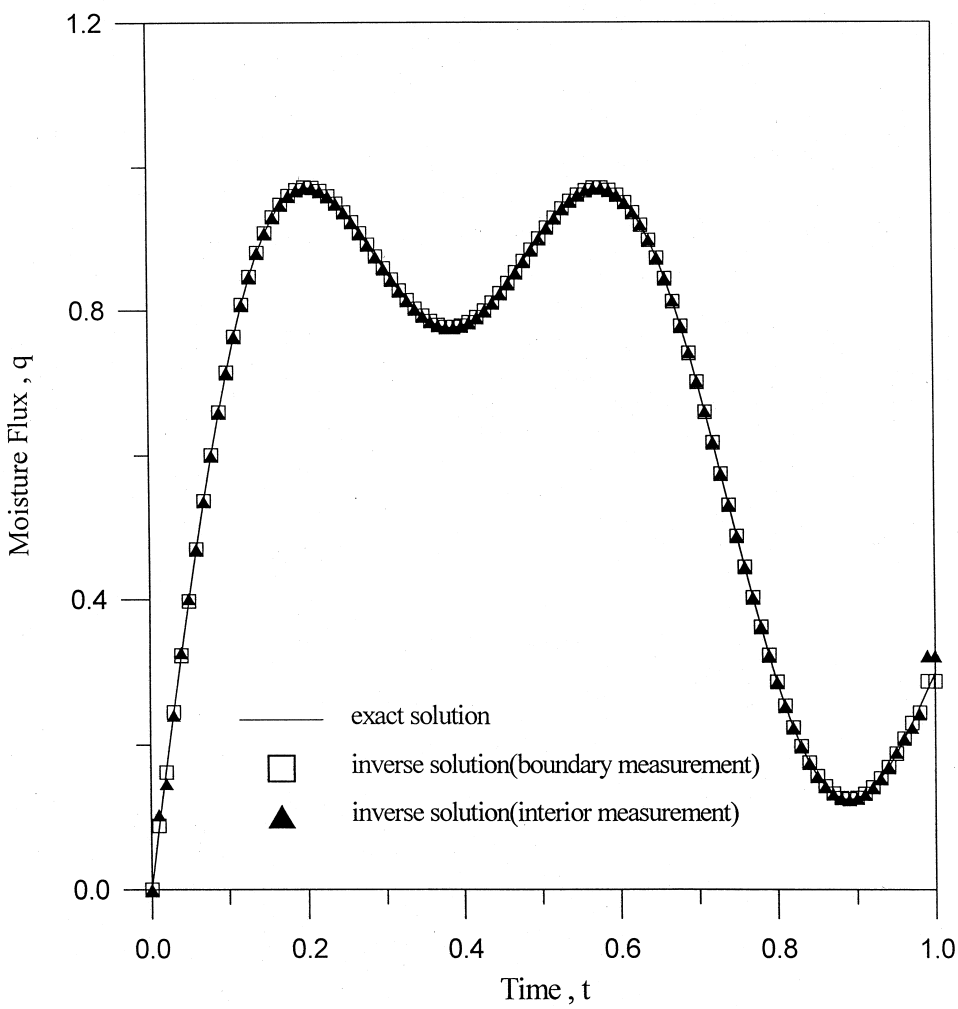

Fig. 2. The estimated moisture flux at initial guesses $q^0 = 0.4$ and 30th iteration with $\sigma = 0.0$.

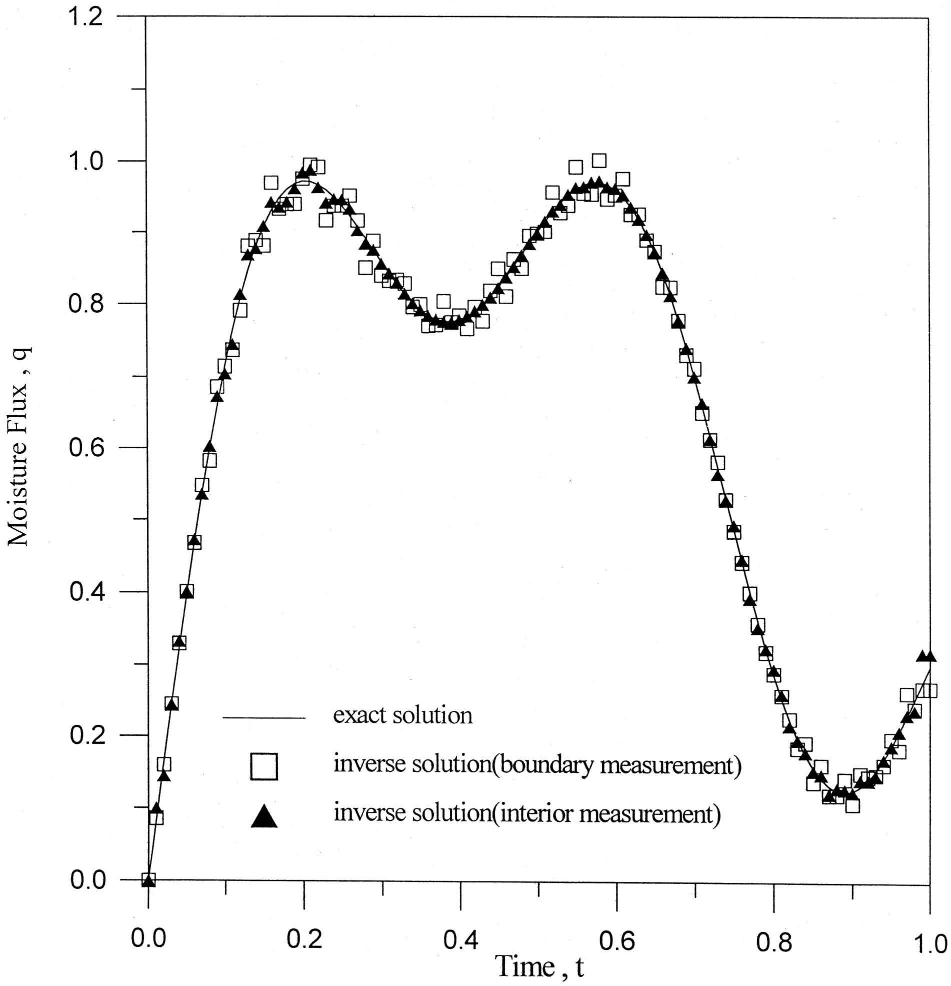

Fig. 3. The estimated moisture flux at initial guesses $q^0 = 0.4$ and 30th iteration with $\sigma = 0.005$.

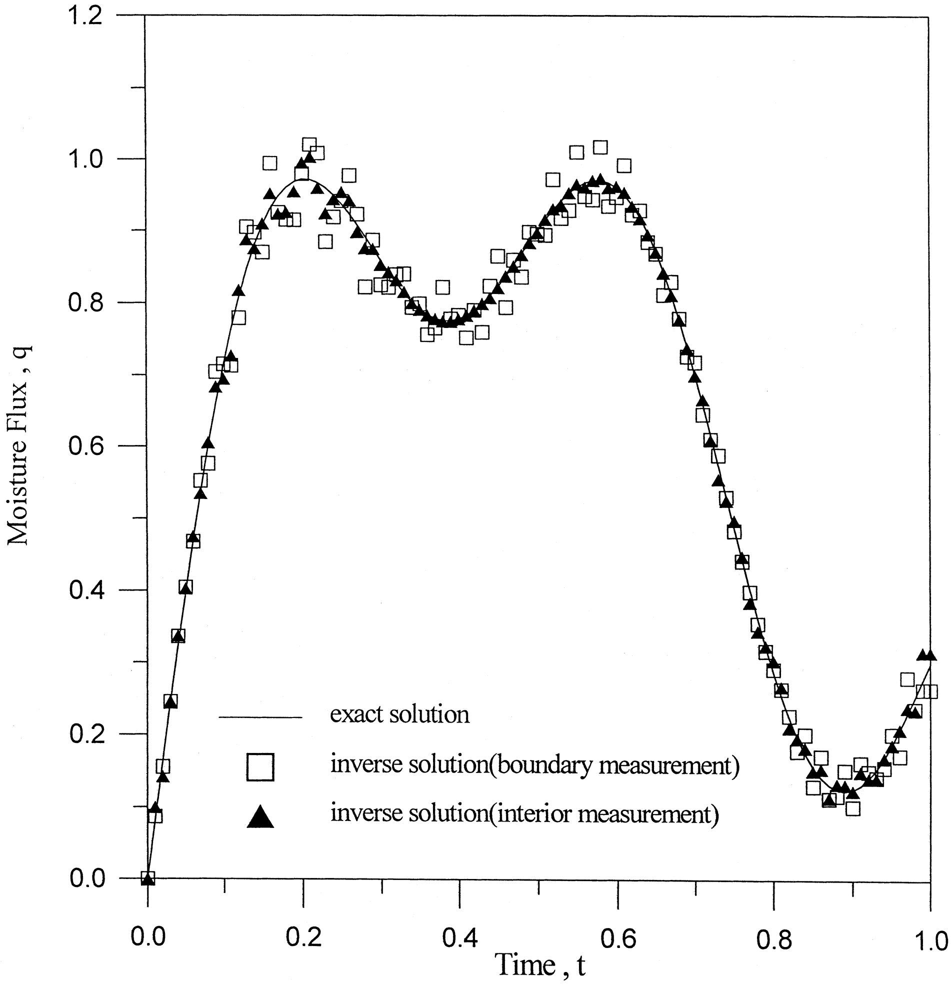

Fig. 4. The estimated moisture flux at initial guesses $q^0 = 0.4$ and 30th iteration with $\sigma = 0.01$.

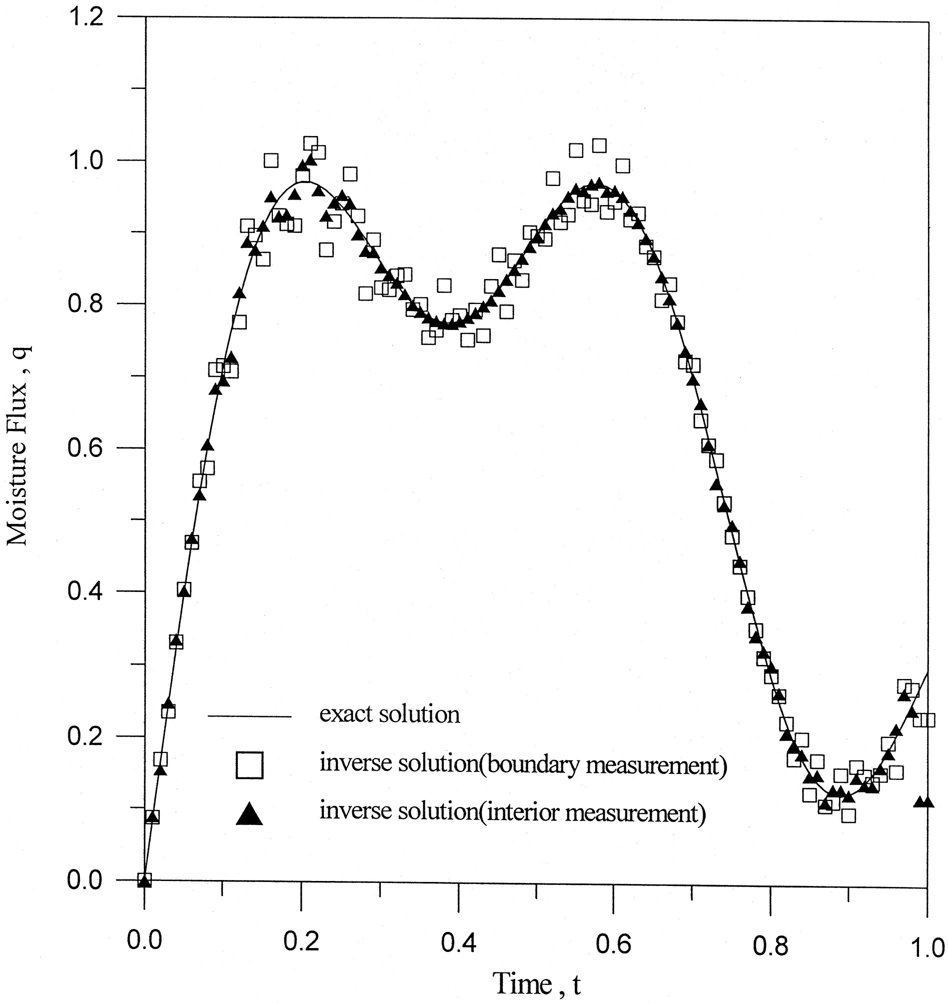

Fig. 5. The estimated moisture flux at initial guesses $q^0 = 10^{-3}$ and 30th iteration with $\sigma = 0.01$.

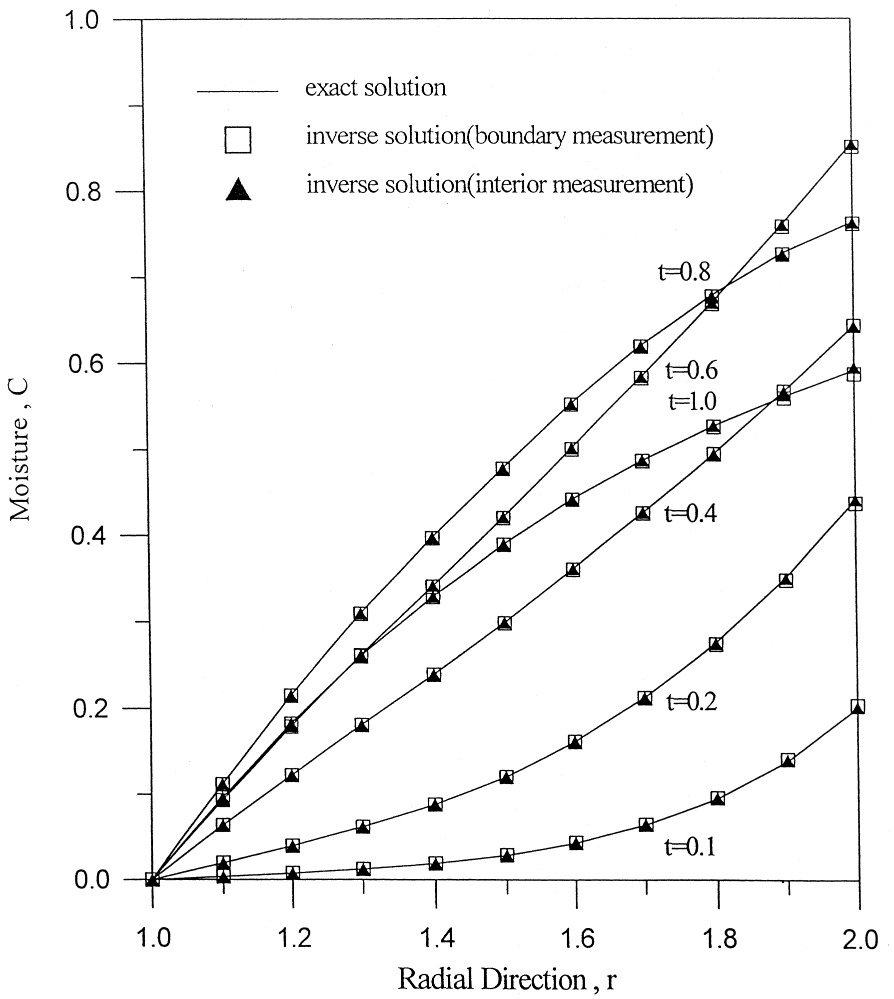

Fig. 6. The estimated moisture distributions at initial guesses $q^0 = 0.4$ and 30th iteration with $\sigma = 0.01$ for various times.

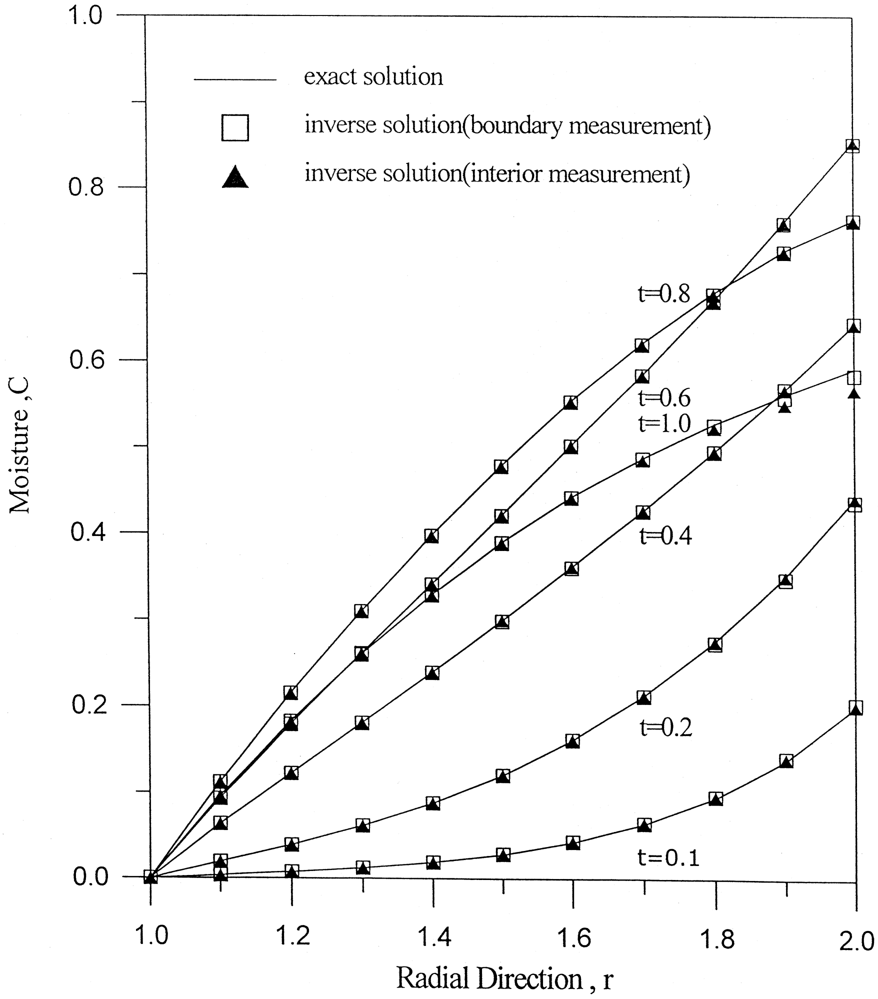

Fig. 7. The estimated moisture distributions at initial guesses $q^0 = 10^{-3}$ and 30th iteration with $\sigma = 0.01$ for various times.

transport in an infinitely long annular cylinder. In this method, the moisture distributions along the radial direction for various times can be estimated by measuring a boundary or an interior temperature. Numerical results confirm that the method proposed herein can accurately estimate the moisture distribution even for the problem involving the error of temperature measurement, regardless of whether measured point is in the interior or on the boundary. The results also indicate that the method by measuring the temperature at the two-layer interface can reduce the sensitivity for measured errors.

Moreover, the fact that experimentally determining the moisture distribution is rather difficult accounts for why the proposed method is highly promising for designing a moisture sensor for some solids.

Appendix A

The dimensionless analytical solutions of temperature T and moisture concentration C for the two-layer cylinder are given as follows [2]:

$$T_j(r,t) = \frac{D[f_{1j}(t) - f_{2j}(t)]}{L(S_1+S_2)} \cdot \left[\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\delta_{1j} + \delta_{2j}\right]$$

$$- \frac{\pi}{L(S_1+S_2)}$$

$$\cdot \sum_{i=1}^{2} \sum_{n=1}^{\infty} \frac{(-1)^{i+1}[J_0^2(\beta_{nj}a)\delta_{1j} + J_0(\beta_{nj}a)J_{j-1}(\beta_{nj}b)\delta_{2j}]R_{0j}(\beta_{nj}r)}{J_0^2(\beta_{nj}a) - J_{j-1}^2(\beta_{nj}b)}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_{nj}^2(t-\tau)/d_i^2} f'_{ij}(\tau)\,\mathrm{d}\tau - \frac{\pi D}{L(S_1+S_2)}$$

$$\cdot\sum_{i=1}^{2}\sum_{n=1}^{\infty}\frac{(-1)^{i+1}J_0^2(\beta_{n2}r_2)R_{02}(\beta_{N2}r)}{\beta_{n2}[J_0^2(\beta_{n2}r_2)-J_1^2(\beta_{2n}r_3)]}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_{n2}^2(t-\tau)/d_i^2} q'(\tau)\,\mathrm{d}\tau\cdot\delta_{2j}\quad j=1,2 \tag{A1}$$

$$C_j(r,t)=\frac{S_2f_{1j}(t)+S_1f_{2j}(t)}{S_1+S_2}\cdot\left[\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\delta_{1j}+\delta_{2j}\right]$$

$$-\frac{\pi}{S_1+S_2)}$$

$$\cdot\sum_{i=1}^{2}\sum_{n=1}^{\infty}\frac{[J_0^2(\beta_{nj}a)\delta_{1j}+J_0(\beta_{nj}a)J_{j-1}(\beta_{nj}b)\delta_{2j}]R_{0j}(\beta_{nj}r)S_\alpha}{J_0^2(\beta_{nj}a)-J_{j-1}^2(\beta_{nj}b)}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_{nj}^2(t-\tau)/d_i^2} f'_{ij}(\tau)\,\mathrm{d}\tau + q(t)r_3\ln(r/r_2) - \frac{\pi}{S_1+S_2}$$

$$\cdot\sum_{i=1}^{2}\sum_{n=1}^{\infty}\frac{J_0^2(\beta_{n2}r_2)R_{02}(\beta_{n2}r)S_\alpha}{\beta_{n2}[J_0^2(\beta_{n2}r_2)-J_1^2(\beta_{2n}r_3)]}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_{n2}^2(t-\tau)/d_i^2} q'(\tau)\,\mathrm{d}\tau\cdot\delta_{2j}\quad j=1,2 \tag{A2}$$

where

$$S_j=\{(-1)^{j+1}(L/D)\pm[(1-L/D)^2+4v\lambda L/D]^{1/2}\}D/2L \qquad j=1,2$$

$$S_\alpha=S_2 \quad \text{for } i=1$$

$$S_\alpha=S_1 \quad \text{for } i=2$$

$$d_i^2=\frac{D}{L}\cdot\frac{S_i+(-1)^i v\lambda}{S_i}=1+(-1)^i S_i \quad i=1,2$$

$$a=r_1 \quad \text{and} \quad b=r_2 \quad \text{for } i=1,2,\quad j=1$$

$$a=r_2 \quad \text{and} \quad b=r_3 \quad \text{for } i=1,2,\quad J=2 \tag{A3}$$

and $R_{0j}=(-1)^j[J_{j-1}(\beta_{nj}b)Y_0(\beta_{nj}r)-Y_{j-1}(\beta_{nj}b)J_0(\beta_{nj}r)]$, $j=1,\ 2$; j_μ and Y_μ are the Bessel functions of the first and second kind of order μ, respectively; in which the eigenvalue β_{nj} satisfies the relation $J_{j-1}(\beta_{nj}b)Y_0(\beta_{nj}a)-Y_{j-1}(\beta_{nj}b)J_0(\beta_{nj}a)=0, j=1, 2$, and $n=1, 2, 3, \ldots$; the interfacial function $f_{ij}(t)$ can be determined by the interfacial conditions.

However, considering a one layer case, the solutions of temperature and moisture concentration in Eqs. (A1) and (A2) can be reduced as follows:

$$T(r,t)=\frac{\pi D}{L(S_1+S_2)}\sum_{i=1}^{2}\sum_{n=1}^{\infty}\frac{(-1)^i J_0^2(\beta_n r_1)R_0(\beta_n r)}{\beta_n[J_0^2(\beta_n r_1)-J_1^2(\beta_n r_3)]}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_n^2(t-\tau)/d_i^2} q'(\tau)\,\mathrm{d}\tau\cdot\delta_{2j} \tag{A4}$$

$$C(r,t)=q(t)r_3\ln\left(\frac{r}{r_1}\right)$$

$$-\frac{\pi}{S_1+S_2}\sum_{i=1}^{2}\sum_{n=1}^{\infty}\frac{J_0^2(\beta_n r_1)R_0(\beta_n r)S_\alpha}{\beta_n[J_0^2(\beta_n r_1)-J_1^2(\beta_n r_3)]}$$

$$\cdot\int_0^t \mathrm{e}^{-\beta_{nj}^2(t-\tau)/d_i^2} q'(\tau)\,\mathrm{d}\tau \tag{A5}$$

where S_j, S_α, and d_i defined as (A3); $R_{0j}=J_1(\beta_n r_3)Y_0(\beta_n r)-Y_1(\beta_n r_3)J_0(\beta_n r)$; the eigenvalue β_n satisfies the relation $J_1(\beta_n r_3)\ Y_0(\beta_n r_1)\ -\ Y_1(\beta_n r_3)\ J_0(\beta_n r_1)=0$, $n=1, 2, 3, \ldots$

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